Tractatus Logico Computatus:
On Language, Certainty and the World
Section One:

Critique of the consensus paradigm

LC Type Logic

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LC Type Logic

1 Introduction to the consensus view or paradigm

For any critter that relies on two or more independent sensory-motor channels for its information about the world, situations frequently arise when two or more competing interpretations exist for a given collection of sensory-motor experiences and each competing interpretation would, if acted upon, trigger a mutually exclusive action. In these situations, whether implicitly or explicitly, a choice must be made. Only one action can be triggered.

Regardless whether the critter is a cat or a dog or a person, and if a person regardless whether s/he reflects upon the 'why', the choice faced by the critter and the ensuing action can be represented with a basic form of what logicians would call an '*If… Then*' (or *If*, *Then*, *Else*) statement.

> *If* the interpretation (or belief) for a collection C of sensory-motor experiences is P
> *Then* Perform Action 1,
> *Else* (i.e., If the interpretation (or belief) for a collection C of sensory-motor experiences is Not P)
> *Perform* Action 2

Consider an emergency room doctor needing to quickly make what could be a life or death decision for a newly arrived patient. Depending on the doctor's interpretation of the patient's problem, for which the doctor has perhaps ten seconds to choose a course of action, the doctor may inject the individual with one of two different drugs. If the doctor's interpretation of the patient's problem is correct, the patient will live to recover. If the doctor's interpretation is incorrect, the patient will die in the next ten minutes.

If only the doctor knew how to *guarantee* a correct or true interpretation. But s/he doesn't; s/he couldn't. To what degree is it even possible? That's the question. And why the concept of 'truth' (and in its strongest form, *certainty*) is one of the core concepts that have been pondered on over the ages.

Yet the notion of 'truth' does not stand alone. The number '3' cannot be true (or false); the universe or God cannot simply be true (in a meaningful way to language). No matter how much you may love the color 'green', by itself, in isolation, the color 'green' cannot be true or false. Truth and falsehood are attributes not of objects, words or concepts, but of assertions, statements, declarative sentences or what are often called 'propositions'[1].

The only way you can come to a robust and stable understanding of 'truth' is to have a robust and stable understanding of propositions. And so, the notion of a 'proposition' originally defined by Aristotle as a statement or assertion that must be true or false (in the exclusive sense of the term 'or'), has been of central concern to philosophers (logicians and

[1] Russell called these (e.g., Truth, belief, want) " propositional functions of two verbs". They later came to be called "propositional attitudes"

mathematicians) ever since. And rightly so; for the notion of a proposition lies at the confluence of three great streams of thought. One stream is concerned with how propositions are created. Irrespective of the complexity of human language (e.g. its surface grammatical form), are there some minimal structures, components or functions that all propositions share (i.e., a single logical form or collection of forms)? And are there specific rules of formation? Here again, Aristotle theorized that propositions had an internal structure. Whether called subject and predicate or (thousands of years later) argument and function (or relationship), a proposition required that something be asserted of something else: The 'asserted of' and the 'asserted about'.

A second stream is concerned with how to test propositions that are given. This goes back at least to the difference between Platonic truth by definition and Aristotelian truth by measurement. More recently, some focused on falsifying[2]; others, on verifying[3]. Some focused more on 'correspondence[4]'; others more on 'coherence[5]'. Regardless of strategy, it meant there was a need to trace a proposition back to its sources - to the so-called facts that determined the truth or falsity of the proposition. Following in the spirit of Leibniz, Frege's desire to create a perspicuous chain of reasoning was one of the motivating forces behind his creation of the Begriffsschrift – the origin of the predicate calculus. This stream of thought later resulted in Frege's distinction between 'sense' and 'referent', as well as Russell's between 'denotation[6]', and 'meaning'

A third stream is concerned with how to reason with and about propositions. Given a collection of propositions (as premises) assumed or demonstrated to be true, by what rules can additional propositions (as conclusions) be generated that are *certain* to have the same degree of truth as the premises? Thus emerged Aristotle's famous syllogism "If all men are mortal and Socrates is a man then Socrates is mortal." What is true for all is certain to be true for some. When applied recursively, this same insight generated Aristotle's classification hierarchies. The Stoics went on to devise rules for combining propositions anticipating what eventually became known as propositional logic. Over two thousand years later, creating inferences from 'all to some' lie at the heart of many industrial software programs and language specifications.

A consensus view of propositions and the surrounding canonical logic began emerging in the late 19th century. For example:

1. Frege's characterization of a proposition[7] in terms of f(x) where 'x' denoted an N-adic argument and 'f' denoted the predicate or what was asserted,

[2] Karl Popper " …"
[3] A.J. Ayer "Language Truth and Logic"
[4] Russell during his logical Atomism years
[5] Bradley and the Idealists; later Neurath.
[6] For example Russell's 1905 article "On Denoting" and his
[7] As regards 1 above, whether in conjunction with an existential quantifier (e.g., "There exists an X such that f(x)"), or a universal quantifier, (e.g., "For all 'x' f(x)"), Hilbert, Peano, Russell, Carnap, Quine and Putnam to name but a few, all used a symbology based on the notion of a predicate 'f' and N-adic argument '(x)' to formally denote and reason about propositions.

2. Russell's theory of descriptions linking natural language to the predicate calculus via systematic descriptions of arguments,
3. Mill's, Frege's and Russell's distinction between denotation or reference and meaning or sense, and its application to substitution rules, and
4. Pierce and Wittgenstein's classification of truth functions

have continued in use until this day[8]

Regardless of whether one further believed in the need to add part-whole, temporal, modal, deontic or epistemic predicates (and arguments) to logic, and regardless of whether one adhered to bivalent truth functional logic or was a proponent of multi-valued logics, by the 1930s[9], certainly, consensus had been reached on the deeper issues of

1. Well formedness as regards propositions, (or sentences or statements or WFF)
2. Equivalence and substitution (a precursor for testing), and
3. Kinds of truth (or truth testing functions)

as well as the even deeper and more philosophical issue of ontological commitments[10].

As regards ontological commitments, the 'X' was understood by the consensus view to refer to an object in the world[11]. That object could be as concrete as a desk or chair. Or it could be as abstract as a number. Two names or descriptions were said to denote the same thing if the same object was referred to by each. This was and is still called extensional equivalence with the approach as a whole being termed "extensional semantics".

In contrast, the "f" was not as well understood within the consensus view. For Frege, the "f" symbolized a 'concept'. Others, like Russell, have likened the "f" to an attribute. Though Aristotle did not use a symbology based on f(x), he was arguably the earliest recorded logician and would have (at least within De Interpretatione) called the 'f' that which was asserted. Regardless of what it was called, the "f" did not appear to refer to an entity in the world in the same way as did the "x".

As regards well formedness, natural or surface language well formedness became an operational proxy for logical well formedness. In principle, any sentence comprising any of a number of grammatical forms such as a noun phrase and a verb phrase was well formed[12]. As such, the expressions "This sentence is false.", and "This theorem is undecidable" were both

[8] Over the years alternate terminology has been introduced and used. As far back as the turn of last century Sheffer used the term 'propositional forms' instead of propositional function. Quine spoke of 'sentential functions' instead of propositional functions. Some , like Lee used R(X,Y,Z) instead of F(X,Y,Z) and spoke of relations instead of predicates. See for example "Symbolic Logic" Harold Lee Random House 1961

[9] There was no specific day. But the 1927 second edition of Principia Mathematica with its inclusion of Russell's logical atomism that he took from Wittgenstein is a good marker. In any event the key notions of well formedness, rules of equivalence and sentential connectives were reasonably stable by then.

[10] No attempt is being made here to deny the myriad initiatives in so-called deviant logics. Simply, they typically represent an alternative view on or additional structure added to some aspect of the consensus view.

[11] Certainly there was also the substitutional view. Though Haack considers the referential to be the consensus.

[12] In addition to Verb phrase(Noun phrase), there are also Adjectival phrase(Noun phrase) and Adverbial phrase(Verb phrase) to name but a few.

well formed propositions. Within this view of well formedness, individual propositions were treated as independent logical atoms: both truth functionally independent (e.g., P's and Q's could be related in any possible way) and semantically independent (e.g., the meaning of any P is independent of the meaning of any other P or Q).

As regards kinds of truth, the consensus view, regardless of terminology used and where/when it is grounded (e.g. Plato, Aristotle, Bacon, Descartes, Hume, Kant,…Frege, Russell…..), is that the character of the proposition, its kind of truth (e.g., whether more empirical or definitional) is a function of the proposition (or collection of propositions in a Quinian sense).

And finally, as regards equivalence and substitution, and beginning with Frege if not Mill, the notion of referent- as in the referent of 'x' - serves as the basis for expressions of identity. A collection of names or descriptions are equivalent if they refer to or denote the same object in the world. Sameness of reference then serves as the basis for valid substitutions. Within a proposition, one sub expression, (e.g., a name or description) can be substituted for another if they each refer to the same object in the world.

The consensus view (or major aspects therein) was not without its critiques especially from within by the various philosophers, logicians and mathematicians who created it. The ink was not dry on the consensus view before problems emerged – big problems in the form of paradox and other inconsistencies.

Whether looking at Russell's letter to Frege and Frege's response, or Russell's letters to Lady Ottoline or Wittgenstein's Tractatus, his notebooks from the 1930s or the Investigations, it's clear that the key players were aware of the holes in their foundations. For example, both Frege and Russell were aware of problems of identity and substitution in the predicate calculus, and Russell discovered and passed on to Frege the problems with the predication of all members of a class or set, especially when there were dependencies between the act of determining that an object belonged in the set, and the set itself.

Moreover they also wrote about the attributes they would like to see in a better set of foundations. Russell and Wittgenstein for example both wanted a fully perspicuous notation for logic, one that showed the reasons why certain actions permitted in the canonical paradigm were non-sensical.

Although the past sixty years have witnessed an economic explosion of highly successful propositional logic-based applications in the area of computing (e.g., logic circuits) and software technology (e.g., all Relationally-based data management software, not just specialized AI applications built from Prolog or Datalog), the foundational holes discovered, researched and left open by Frege, Russell and Wittgenstein remain.

The impact of foundational holes is felt not with bread and butter truth functions that seem to work quite fine, and within which no paradoxes have ever been found, but rather within the emerging fields of knowledge management and natural language representation that operate

over more heterogeneous and multileveled domains. And here there are many problems including:

- For knowledge management
  - How to represent, reason or infer across multiple levels of abstraction or aggregation, especially given
    - Inconsistencies, uncertainties and vagueness
  - How to merge qualitative reasoning about words with quantitative reasoning about numbers
  - How to combine verbal reasoning with non-verbal reasoning
  - How to manage beliefs when there are multiple conflicting sources of information for what should be the same proposition
  - How to distinguish incorrectly formed propositions from well formed ones
    - How to process incorrectly formed propositions that are in a larger group of well formed propositions being processed with a single aggregate function, and
- For natural language representation
  - Since not all grammatically well formed sentences convey information and many grammatically ill formed collections of words do convey information, what exactly are the criteria for successful communication, and
  - How can linguistic artifacts (e.g., noun phrase, verb phrase, preposition…) be merged or fused with knowledge artifacts (both mathematical and 'empirical') so that parsing a sentence becomes synonymous with mapping it to a unique location in knowledge space

Of course the foundational holes are a big problem for Logic itself. Some of Logic's biggest problems include

- The Liar paradox
- The set of all non-self membered sets paradox
- Substitution failures
  - If meaning is extensional and two terms have the same extension, how can substitution between terms that have an identical reference alter the truth value of an expression?
- Truth gaps and gluts
  - Keeping the middle excluded - allowing for only P XOR !P - works incredibly well for many real world applications of math and logic. Yet there is something appealing to the notion of truth in degree whether manifested thru multi-valued truth functions or fuzzy predicates or the embrace of contradictions as true. How can these seemingly global and mutually inconsistent approaches be reconciled? From what higher dimensional space can these logics be seen as lower dimensional projections?
- Truth itself

- There are so many different approaches such as Realist, Anti-realist, and Deflationist. Is there any way to reconcile them?
- None of the popular approaches can account for paradigm change. When should whatever is currently serving as a standard for truth be dethroned by something new? And,

Solving, or at least proffering solutions to these problems, can only result from a deep understanding of their causes. Since the causes are embedded in the very fabric of logic - its consensus paradigm, what follows, therefore, is a succinct critique of that paradigm[13], consistent I believe with what at least Russell, Wittgenstein and Toulmin would have acknowledged. It is intended to whet the reader's appetite for a new paradigm that can provide a single clean solution to the variety of old paradigm problems.

The remainder of the paper is composed of four additional sections.

- Section two is an introduction to a new paradigm for logic, one built on the combined principles of cognitive processing with extensible types that have both logical and physical characteristics.
- Section three is a formal model of a subset of LC Type Logic that is restricted to Boolean and Categorical types. The reason for this restriction is that Booleans and Categoricals are sufficient to reconstruct both Classical and non-Classical logics.
- Section four is a formal model of (still a subset) of LC Type Logic that defines more complex types such as integers, rationals and irrationals, various forms of hierarchies, and of network and graph types.
- Section five completes the formal model with the introduction of feelings and emotions as forms of representation

All the problems mentioned above, and more, will be solved through the exposition of the formal model.

[13] The purpose of the critique is to 1) highlight a sufficient number of deep problems with the consensus view to warrant the reader's interest in considering the proffered solution in the form of an alternative paradigm that follows and 2) identify specific problems that the proffered solution will need to show that it can solve in order to be taken seriously. The critique is not exhaustive. Most of the criticisms have been made by and are herein attributed to others.

2 Critique of canonical logic's consensus paradigm

2.1 Ontological commitments for the 'x' in f(x)

The external object-centered paradigm of canonical logic did not begin with Frege. It began at least 2200 years earlier with Aristotle when he introduced a simple ontology comprised of substances and properties and a referential approach to language where grammatical subjects referred to substances and grammatical predicates referred to properties.

While Frege's function-argument notation did circumvent the grammatical bias towards unary arguments, it did not escape the object-centered paradigm within which it was formulated. On the contrary, the introduction of quantification only served to highlight canonical logic's ontological commitedness to predicable external objects. The 'x' in f(x) needed to stand for something in the world: planets, featherless bipeds, trees, whatever[14].

Words were associated with collections of objects in the world - their extension[15]. The concept "tall persons living in Cambridge MA" denoted a certain collection of persons in the world. The proper name "Empire State Building" denoted a specific building in the world. It naturally followed that two concepts were extensionally equivalent if they denoted the same object or collection of objects in the world.

The most popular view of empirical truth (e.g., Tarskian correspondence "Snow is white" is true IFF *snow is white*), laid easily over Frege's extensional semantics and the supposed existential certainty of singular objects. If anything is certain it's that "This *is;* this *must be* the Empire State Building."

Yet it appeared possible for two apparently different collections of words to denote the same object or collection of objects. Did not the term 'evening star' denote the same object as the term 'morning star'? Did not the proper name Napoleon denote the same object in the world as the description 'the victor of Austerlitz'?. So the distinction between sense and referent was born. The referent was the extension; the sense was not well defined but was correlated with the obvious difference between the sequences of tokens which nonetheless seemed to share the same referent. (In economic terms, sense could be thought of as a residual; in mathematics, an error term. Philosophically it was considered a part of the intension of the term or terms)

Though it is tempting to suggest that we need to ground logic and truth in referents, extensions or the 'external world' and while they are implicit in extensional definitions of identity, neither meaning nor

[14] Of course, as Russell discovered, locating logical connectives in the world was more complicated.

[15] Ontological footnote
Anything can be an entity. The smallest subatomic particle cld be an entity; our known universe cld be an entity. However, given something specific as an entity, not anything (else) can be

- a part of that entity or
- a group of entities

that includes the given entity. Moreover, anything specific can simultaneously, without internal contradiction be represented as a part of an entity, a single entity, or a group of entities.

So it makes no sense to opine about whether the world is 'one' or a single fact or entity within which many interrelated things co-exist, or whether the world comprises distinct objects. Both are limited views of a common whole.

any action or computation that might follow from an understanding or processing of a proposition requires any specification of a referent in the external world. Nor is such a specification ever possible. For like it or not, we – in the sense of our individual cognitive processes- are prisoners of our own sensory-motor apparatus. My knowledge of the so-called referent of the term 'Empire State Building" is still limited to a collection of sensory motor experiences e.g., visual patterns representing some global positioning device that indicates my presence in the vicinity of the empire state building; some visual patterns consistent with those I've experienced before though coming from a book where there were words indicating the picture was a picture of the Empire State Building; perhaps tactile and inner ear patterns indicative of climbing followed by visual patterns consistent with viewing buildings from above. As rich as these sensory-motor experiences are, they are all happening inside each one of us. There's no way to be aware of the external world outside the mind except indirectly from within. Logic should embrace and not deny this fact.

We are not claiming there are no externally grounded referents. There may well be an external world. Rather there is no way to know what exactly is the source and/or target of our physical representations. If a grounding for certainty is the goal, our so-called external world is the worst place to drop anchor. Its depths are without bounds. Though we may live as if we knew the external world and the objects it contains, when push comes to shove, we are forced to admit that its very existence is an eternal hypothesis. So philosophical agnosticism (neither skepticism nor blind faith) is called for in this regard.

The drawing of boundaries or distinctions that produce individuation starts in the mind. The world does not come pre-packaged. Better therefore to base logical notions of identity on correlations between physical representations whether between multiple sensory-motor pathways for a single hypothesized referent for a single individual

> (it looks like camembert – there exists a visual pattern consistent with my definition for a cheese called camembert- , it smells like camembert – there exists an olfactory pattern consistent with my definition for camembert and it tastes like camembert – there exists a gustatory pattern consistent with my definition for camembert),

or for the same sensory-motor pathway between individuals

> (Bob who from audio and visual patterns I believe is standing next to me is experiencing a visual pattern consistent with his definition of what he calls snow falling and so am I).

One could further postulate that Bob and I have similar systems of physical sensory-motor representations and thus (by assuming some flavor of the law of self-identity), whatever is causing Bob's experienced physical representations is the same as what is causing mine…whatever that might be…"in reality".

And so the notion of referent, such a seemingly modern, logical, analytic and objective construct is really no different than the classic philosopher's (e.g., Kant's, Hegel's, Aristotle's or Plato's) mystical notions of 'being', 'becoming', or 'things-in-themselves'; that which is eternally unknowable. It's a great source of philosophical wonder; but a black hole of epistemology. And an impossible place from which to extract or grow any model at all much less a model of logic and truth.

2.2 Well formedness

Aristotle's original notion of a proposition was defined as an assertion that must be true or false. It was presumed that the assertion was meaningful and that it was meaningful because it was well formed. But what exactly was a well formed assertion or proposition (i.e., what is the membership function for the

class of all well formed propositions)? It's tempting to assert that any well formed declarative sentence in a 'natural language' such as English denotes one or more propositions[16]. But then what to make of seemingly nonsensical adjectives and adverbs as in Chomsky's well known example phrase from his 'Syntactic Structures' (e.g., "Green ideas sleep furiously.")? It would appear that natural language and many formal languages[17] allows the construction of sentences that common sense would dictate are meaningless and have no truth value.

A logician might respond that it's the job of the semanticist to make the contingent restrictions for which predicates can apply to which arguments. In other words, there's nothing necessary about the contingent fact that ideas have no color or sleep can't be performed emotively. This is a restriction that is layered on after the fact so to speak. So there is nothing logically ill formed about declarative sentences such as

"Green ideas sleep furiously", or
"Invisible sounds can only be heard if they are blue."

A logician could even put them in f(x) form as follows:

Sleep furiously (Green ideas)
Can only be heard if they are blue (Invisible sounds)

A creative semanticist could come up with some interpretation within which the sentence is meaningful. While such a retort might work on *green ideas* and *invisible sounds*, what about declarative sentences that aren't just meaningless but seemingly contradictory such as the following:

The colorless liquid was blue

In fact, a logician would say there's no problem here either. Given the standard interpretations of the various symbols, the sentence is false – by definition. And besides, it's not up to the logician to set the meanings for symbols. That, again, is the job of the semanticist.

So perhaps any grammatically well formed declarative sentence is logically well formed. And it is always a matter of contingent interpretations of the sentence symbols that further determines whether the sentence is meaningless (and has no truth value) or meaningful (and has one).

For example, working with a surface language one could add in general grammatical distinctions (that would apply to a wide variety of sentence structures) such as

1. Verb phrase(Noun phrase)….
    a. sleep furiously(Green ideas)
2. Adjectival phrase(Noun phrase)…
    a. Blue(Book)
3. Adverbial phrase(Verb phrase)……

[16] We say one or more because most sentences (e.g., 'The sun is shining', 'That person is my brother', 'The movie theatre is across the street from the diner') only individuate their argument(s) within a locally defined context. So each time the same assertion is made within a different context, it counts as a different proposition with an independent truth value.

[17] [Both the rules of Chomsky/Montague linguistic transformations, and the symbolic languages of formal logic (a la Russell/Zermelo-Franken) allow construction of sentences that are well-formed according to the rules, but nonsensical or paradoxical according to their semantic content and common sense meanings.]

a. Heading south(Flying geese)

Any one of the above combinations forms a grammatically well formed assertion. And with the traditional symbol interpretations, sentence 1.a would be logically meaningless while sentences 2.a and 3.a are logically meaningful well formed propositions with truth values.

However, and here comes the real problem, regardless of grammatical form and regardless of contingent symbol interpretations, not all logical predicates can be predicated of logical arguments. Some logical predicates[18] can only be applied to *already well-formed propositions*. For example, the predicates "Belief" and "Truth test result or value"[19]. Moreover, these predicates are required for any formal logical system. After all, what good would such a logical system be if it did not support any kind of truth or belief functions? So logically speaking (i.e., in all cases for any working logical system), there is a need to distinguish between predicates that apply to arguments (parts of propositions) and predicates that apply to well formed propositions in their entirety (typically called propositional attitudes).

Consider the liar paradox in its Russellian form:

'This sentence is false'

The sentence is grammatically well formed -in f(x) terms it reads verb phrase(noun phrase)-, but internally contradictory in that it appears to be true if false and false if true – hence, paradoxical. Although the classical view of the problem was that it stems from the self referential nature of the sentence[20] and so Russell's solution was to create a hierarchy of sentence types, the problem as shown by Barwise and Echtemendy in their book "The Liar Paradox"[21] has nothing to do with self reference, per se.

For example, "This sentence has five words." is no less self referential than "This sentence is false". But it is true, not paradoxical. (As, "This sentence has six words.", is false; not paradoxical.) Barwise and

[18] Meaning that regardless of the semantic interpretation of the symbols of the language, some symbols would need to denote logical predicates that only apply to propositions.

[19] By treating 'Truth value' and 'Belief' as predicates we are not claiming that they exist as separate predicates from the act of asserting. In other words, to assert that 'the sky is blue' is the same as to assert that 'the sky is blue is believed to be true' and so if 'the sky is blue' maps to P so too does 'the sky is blue is true' and 'The sky is blue is believed to be true' map to P. However, the fact that an assertion is originally given as true and believed does not mean that additional tests of truth or belief cannot be meaningfully predicated. If after receiving the assertion 'the sky is blue' the receiver then looks outside and sees that in fact the sky is blue, that second independent test of the originally given assertion would be represented as a second independent assertion of truth. For example, 'Yes it is blue.', which does not reduce to the originally stated assertion 'The sky is blue'. Mapping to propositional variables, it would not be correct to simply say P is true. This is because the original assertion –regardless of the nature of the assertion itself (e.g., be it about snow or the author of Waverly or featherless bipeds etc..) - came embedded with the notion of 'being asserted as true' whereas the second assertion is specifically an assertion of truth about P which -as with all assertions – does come embedded with its own notion of 'being asserted as true'. So given indexicals to identify for any propositional variable the argument and predicate, the original assertion 'P' could be any valid assertion about anything . While Q must take P as its argument, and must make an assertion about P's truth value. Using subscripts for both indexicals this could be represented as follows: P_{xf} , Q_{ptrue} .

[20] Need attribution here

[21] Barwise, Echtemendy "The Liar Paradox" pps…

Echtemendy made the case that the flip flopping truth values of the Liar reflected flip flopping contexts within which the terms 'this sentence' were to be interpreted. And within each interpreted context, the meaning and truth value of 'this sentence is false' is stable.

As they noted, adding explicit dimensions that are otherwise implicit within the argument structure of a proposition is one way to resolve what otherwise appears as conflict or inconsistency. And while Barwise and Echtemendy offered great examples of where this does solve a problem such as two individuals having a conversation at a moment in time and yet being unable to resolve what time it is. And this, because they are talking on a phone from different time zones. Once a time zone dimension is added to the argument structure, there is a single time for their call that can be represented (based on the time zone of each participant), in any number of different local times.

Their insightful analysis of hypersets and of Russellian versus Austinian semantics notwithstanding, Barwise and Echtemendy fell into the same trap that caught Frege and Russell before. Namely the trap of thinking that just because a sentence is grammatically well formed (e.g., 'This sentence is false' has a noun phrase verb phrase structure) that it denoted one or more well formed propositions.

Rather, the problem is one of well formedness. The predicate 'false' applies to propositions, not terms. "The book is brown is false." is just fine as a proposition-denoting sentence that includes the term 'false' as a predicate. Because 'false' applies to each evaluation of the proposition "The book is brown.". In contrast, the sentence "Blue is false." does not denote any proposition. The predicate 'false' cannot be evaluated with respect to a term such as 'blue'.

In the consensus analysis of "This sentence is false", (e.g., Russellian, modern classical and modern non-classical), the term 'this sentence' acts as a pointer to one or more sentences that get substituted in for the pointer. After substitution, the resulting proposition is evaluated and its truth value recorded.

The error with the consensus view is that it jumps the gun in assigning a propositional variable and one or more truth values to the paradoxical sentence. The reason the consensus approach is problematical is that at no point in the substitution process does the sentence ever denote a truth value- bearing proposition. Since that possibility was brought up by Barwise and Echtemendy[22] but then rejected for lack of evidence, we now meet their challenge to "offer an explanation for the failure of the Liar sentence to express a claim one that is either true or false"[23], thereby demonstrating that well formedness **is** the fundamental issue. Let's begin with a well formed self referential sentence to see how things normally work.

Consider the process of trying to compile the sentence 'This sentence has five words' into a form that denotes a truth-testable proposition.

Step 1: Receive string in context ; parse into words 'This sentence has five words'

Step 2: Assign each word a role as argument or predicate

- Has five words(this sentence)

Step 3: Test for the possible presence of pointers

[22] "The Liar Paradox" pps 13, 14
[23] Ibid page 14

- Discover that the two words 'this sentence' in the argument could[24] constitute a pointer.

Step 4: Check whether the candidate pointers discovered in step 3 are the output of a prior dereferencing and check whether the words match the form required for the predicate operator. There are three cases to process the results:

Case 1: If the words are not the result of a prior dereferencing then substitute for the pointers the string that is what the pointers point to and return with that string to step one

Case 2: If the words are the result of a prior dereferencing and the words match the form required for the predicate operator then run/execute the predicate operator on the argument

Case 3: If the words are the result of a prior dereferencing but they do not match the form required for the predicate operator then substitute for the pointers the string that is what the pointers point to and return with that string to step one

Given the output of Step 3 above, step four triggers case 1. And so

Case 1 repeating Step 1: Receive string in context ; parse into words
> has five words ('This sentence has five words')
Case 1 repeating step 2 Assign each word a role as argument or predicate
> has five words ('has five words(this sentence)')
Case 1 repeating Step 3: Test for the possible presence of pointers
- Discover that the two words 'this sentence' in the argument could constitute a pointer.

Case 1 repeating Step 4: Check whether the candidate pointers discovered in step 3 are the output of a prior dereferencing and check whether the words match the form required for the predicate operator .
> Discover the candidate pointers are the output of a prior dereferencing and the words do match the form required for the predicate operator

Case 1 step 4 triggering Case 2: If the words are the result of a prior dereferencing and the words match the form required for the predicate operator then run/execute the predicate operator on the argument
- Execute the expression 'has five words' by
  - Counting the words in parentheses '(has five words(this sentence))' which equals five
  - Comparing the number of words with the number 'five' which equals 'sameness'
  - So concluding the assertion is true.

Consider now the using the exact same process to try to compile the sentence 'This sentence is false' into a form that denotes a truth-testable proposition.

Step 1: Receive string in context ; parse into words 'This sentence is false'

Step 2: Assign each word a role as argument or predicate
- Is false(this sentence)

Step 3: Test for the possible presence of pointers

[24] We use the qualifier 'could' because in most machine languages when a term is used as a pointer that fact is explicit in the expression whether through some hardwired mechanism or special syntax

- Discover that the two words 'this sentence' in the argument could constitute a pointer.

Step 4: Check whether the candidate pointers discovered in step 3 are the output of a prior dereferencing and check whether the words match the form required for the predicate operator. There are three cases to process the results:

Case 1: If the words are not the result of a prior dereferencing then substitute for the pointers the string that is what the pointers point to and return with that string to step one

Case 2: If the words are the result of a prior dereferencing and the words match the form required for the predicate operator then run/execute the predicate operator on the argument

Case 3: If the words are the result of a prior dereferencing but they do not match the form required for the predicate operator then substitute for the pointers the string that is what the pointers point to and return with that string to step one

Given the output of Step 3 above, step four triggers case 1. And so

Case 1 repeating Step 1: Receive string in context ; parse into words
> is false ('This sentence is false')
Case 1 repeating step 2 Assign each word a role as argument or predicate
> is false ('is false(this sentence)')
Case 1 repeating Step 3: Test for the possible presence of pointers
- Discover that the two words 'this sentence' in the argument could constitute a pointer.

Case 1 repeating Step 4: Check whether the candidate pointers discovered in step 3 are the output of a prior dereferencing and check whether the words match the form required for the predicate operator .
- Discover the candidate pointers are the output of a prior dereferencing and the words do not match the form required for the predicate operator
    - This is because unlike "normal predicates" like 'color' or 'count of words' the predicate 'is false' requires an extant proposition or truth testable assertion for its argument (not the promise of a proposition by substitution), and (is false(this sentence)) does not constitute a truth testable assertion. As expressed, the words constituting the inner argument 'this sentence' do not form a proposition. No more so than the words 'big blue' or 'real happy' or even 'this toy'. Only when the terms 'this sentence' are replaced by a genuine proposition (e.g., the car is blue) can the inner predicate 'is false' be evaluated of the words as they are and then any outer predicate requiring a proposition for its argument can then be evaluated.

Case 1 step 4 triggering Case 3: If the words are the result of a prior dereferencing and the words do not match the form required for the predicate operator then substitute for the pointer 'this sentence' the string that is what the pointer points to and return with that string to step one

Case 1 Case 3 repeating Step 1: Receive string in context ; parse into words
> is false(is false ('This sentence is false'))
Case 1 Case 3 repeating step 2 Assign each word a role as argument or predicate
> is false (is false('is false(this sentence)'))
Case 1 Case 3 repeating Step 3: Test for the possible presence of pointers
- Discover that the two words 'this sentence' in the innermost argument could constitute a pointer.

Case 1 Case 3 repeating Step 4: Check whether the candidate pointers discovered in step 3 are the output of a prior dereferencing and check whether the words match the form required for the predicate operator .

- Discover the candidate pointers are the output of a prior dereferencing and the words do not match the form required for the predicate operator
    - This is because the predicate 'is false' requires an extant proposition or truth testable assertion for its argument (not the promise of a proposition by substitution), and (is false(this sentence)) does not constitute a truth testable assertion. As expressed, the words constituting the innermost argument 'this sentence' do not form a proposition. No more so than the words 'big blue' or 'real happy' or even 'this toy'. Only when the terms 'this sentence' are replaced by a genuine proposition (e.g., the car is blue) can the innermost predicate 'is false' be evaluated of the words as they are and then any outer predicate requiring a proposition for its argument can then be evaluated.

Case 1 Case 3 repeating step 4 repeat triggering Case 3: If the words are the result of a prior dereferencing and the words do not match the form required for the predicate operator then substitute for the pointer 'this sentence' the string that is what the pointer points to and return with that string to step one …… The process will loop indefinitely hitting Case 3 in step four each time.

So when the process of predication is examined in detail, the classically paradoxical sentence 'this sentence is false' is seen as an infinite loop, not of contradictory true/false assertions but of attempts to substitute an assertion for a pointer to what is supposed to be, but in fact is not, an assertion. The substitution process never comes to a halt; no assertion is ever found. The number of nested pointer substitutions grows without end. The process of finding an argument that is well formed relative to the predicate never terminates. No truth value is ever produced. The sentence is thus meaningless; and for entirely understandable reasons. In this sense, the Liar is similar to Zeno's paradox of motion where once it is understood that the supposed impossibility of motion is caused not by any physical difficulty but rather as a result of looking at successive motion in terms of increasingly smaller time increments, (where clearly as those time increments approach zero, the distance traveled approaches zero as well), the paradox disappears; we escape the fly bottle. Of course a good compiler (or run time execution machinery) would recognize the presence of a repeating loop, flag the sentence as illegitimate and not waste any resources trying to locate a nonexistent proposition.

So the consensus paradigm that uses grammatical well formedness as a proxy for logical well formedness and further lacks any mechanical procedure for distinguishing between grammatically well formed sentences that do and do not constitute one or more logically well formed propositions leads to contradiction and paradox[25].

[25] Of course there is widespread recognition of the paradoxical sentences. But the typical response has been to internalize all of the deviant sentences (i.e., logically ill-formed sentences that do not constitute propositions and have no truth value) and expand the set of logical truth values thereby necessitating an expansion and reinterpretation of truth functional connectives. While there is nothing wrong with creating a multi-valued logic, it is not necessary (see Shavel Thomsen 1993). And it is not efficient. All multi valued logics can be reduced to two valued logic with decision procedures for deciding when a collection of tokens does or does not constitute a well formed proposition.

2.3 Logical atoms

Treating individual propositions as logical atoms is also problematical. It would appear to violate the necessary[26] law of self-identity[27]. This is because, classically speaking, a given 'x' can only have one value per time and space for a given attribute. And so assuming a book to be capable of having only one color per space-time[28], a book can be blue or it can be brown. But it cannot be blue and brown. Yet, Brown(Book) and Blue(Book) are both valid atomic propositions which is why they appear to violate the law of self identity. Something's gotta give.

2.1 Definitional versus empirical propositions

Contrary to canonical approaches at divvying up propositions into empirical, definitional (and tautological) categories, (e.g., '2 + 3 = 5' and 'A bachelor is an unmarried man' are examples of definitional statements whose truth should carry no error term, while 'The burrito came without guacamole' and 'the window is broken' are examples of empirical statements whose truth needs to carry an error term), the answer to the question of whether a proposition is grounded in fact or definition, has nothing to do with the proposition. Rather the truth character of the proposition is a function of the specific question-answering path by which the proposition was generated. Consider an assertion about the color of a car, say, "The car is green" as an answer to the question "What is the color of the car?". Canonical wisdom would say that the proposition "The car is green." is empirical (dare we say synthetic), because knowing the color requires observation or measurement. But is this so? Not necessarily .

If answering the question "what is the color of the car" triggers an action of looking (Russell's knowledge by acquaintance) – the referent or physical representation of 'car' is the collection of color dots in the region of visual sensor space accessed/touched when a car shape is located. And the proposition produced as an answer, 'The car is green' is definitely empirical.

However if the question is answered by repeating what someone had said (Russell's "knowledge by description"), the referent or physical representation is then the remembered assertion in audio space. And the proposition produced as an answer 'The car is green', has a character determined by how the person who originally said 'the car is green' produced that proposition.

And if the question is answered by applying a rule, say "all cars are green", the referent or physical representation is an expression in definitional space. And the proposition produced as an answer 'The car is green', is definitely definitional.

So, what is a definitional (or analytic) assertion to one person (the car is green because all cars are green) may be empirical (or synthetic) to another (the car is green because it looked green to me). Thus, there is no such thing as a definitional or empirical proposition by virtue of the proposition itself. Because the distinction applies to question answering processes not outputs.

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[26] The term 'necessary' is used here because without the law of self-identity, there could be no definably consistent system of inference AKA logic.

[27] [Aristotle's Principle of non-contradiction (Two Laws in Aristotle: the Law of Self-identity "a = a," or "~(a = ~a)" and ~(*f*(*x*) = ~*f*(x))]

[28] This assumption does not hold in non-classical logics such as Dialetheism.

This is why Mill was perfectly justified grounding arithmetic in external phenomena. His only mistake (a big one) being to not then acknowledge that his arithmetic would always carry some measure of error or doubt as to the output of arithmetic operations. Wittgenstein touched on this notion of pathway as well in his notebooks 32-35. He distinguished between arriving at the answer to an arithmetic problem by performing a calculation versus recalling a previously stored answer.

2.2 Identity, equivalency and substitution

Beginning at least with Frege, (if not with Mill), it was thought that the strongest form of truth was the "sameness of reference" of two (or more) names or descriptions. If two or more names or descriptions had the same referent, the assertion of that sameness carried the same kind of truth as the law of self identity, which is still the most certain of truths. Statements that asserted co-reference became known as identities or identity statements. Moreover statements of identity were generally broken out into two groups: trivial identities and non-trivial or informative identities.

Common examples of trivial identity statements include

Scott is Scott
9=9

Common examples of informative identity statements include

Scott is the author of Waverly
The man with the funny hat is Scott
Richard Nixon is Tricky Dick
Nine is the number of planets.

In each case (both the trivial and the informative), the sentence fragment on the left hand side of the copula is asserted to refer to the same thing as what is referred to by the sentence fragment on the right hand side. And where it is obvious that in the case of trivial identities the symbolic representation on the left hand side is the same as the symbolic representation on the right hand side, this is precisely what is not the case for informative identity statements.

All informative identity statements can be[29] informally cast as a = b meaning simply that the symbolic representations on the two sides of the copula are not the same. This dichotomy between the sameness of reference and the difference in symbolic representation has, since the nineteenth century, been explained in terms of a distinction between connotation and denotation (Mill), sense and reference (Frege), or meaning and denotation (Russell).

Substitution rules were then based on the sameness or identity of reference between the symbolic representation being substituted out and the one being substituted in. Thus for example, 'The man with the funny hat' could be substituted for 'Scott' in the sentence

[29] As is done by Frege, Russell, Wittgenstein, Church…

'Scott is the author of Waverly.' to produce the sentence 'The man with the funny hat is the author of Waverly.'. This is because both 'the man with the funny hat' and 'Scott' refer to or identify the same person.

There are three significant problems with this notion of substitution based on identity of reference. The first is that so-called identity statements only account for some, (and a minority at that of), statements or propositions. Russell distinguished between identity statements and predicational statements[30] and advocated that an ideal logical language should cleanly distinguish between the two uses for the copula[31].

Consider below some commonly occurring statement types[32] including predication, all of which assert (i.e., are capable of being true or false). For each statement an example negation (or alternative statement) is shown indented on the line below.

- identity, of the form "*noun copula definite-noun*" *[The cat is Garfield]*
    - The cat is not Garfield
- class membership, of the form "*noun copula noun*" *[The cat is an animal]*
    - *The cat is a plant*
    - *The cat is a mammal*
- predication, of the form "*noun copula adjective*" *[The cat is furry]*
    - *The cat is bald*

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[30] It is a disgrace to the human race that it has chosen to employ the same word "is" for these two entirely different ideas (predication and identity) - a disgrace which a symbolic logic language of course remedies. (Russell 1919:172)

[31] Some authors have gone so far as to say that copulas are unnecessary. For example, in http://en.wikipedia.org/wiki/E-Prime "Bourland and other advocates also suggest that use of E-Prime leads to a less dogmatic style of language that reduces the possibility of misunderstanding and for conflict.[3] Some languages already treat equivalents of the verb "to be" differently without obvious benefits to their speakers. For instance, Arabic, like Russian, lacks a verb form of "to be" in the present tense. If one wanted to assert, in Arabic, that an apple looks red, one would not literally say "the apple is red", but "the apple red". " LJUDMILA GEIST in her paper "PREDICATION AND EQUATION IN COPULAR SENTENCES: RUSSIAN VS. ENGLISH" http://www.ilg.uni-stuttgart.de/Geist/Publikationen/GeistNancy12_05.pdf also points to the lack of copulas used in certain Russian constructs.

The issue of the role of the copula in conveying meaning can not be separated from the issue of well formedness in general. And well formedness as will be shown in the constructive part of this exposition has both an exchanged form and an executable form. The problem with discussions of the copula is that they show no awareness of this distinction. There are huge degrees of freedom as to what needs to be exchanged in order to convey meaning based on different assumptions of what is already shared between the sender and the receiver. The more that is shared or assumed to be shared, the less that needs to be conveyed. For example, if two partners are in an antiques store looking at a Louis 16th chaise and one of them asks the price and the dealer responds with a high dollar value and one of the partners says 'Ouch.', the meaning is clear. *Ouch* in that context means "The chair is bloody expensive."

[32] These examples were taken from http://en.wikipedia.org/wiki/E-Prime

- auxiliary, of the form "*noun copula verb*" *[The cat is sleeping]*; *[The cat is bitten by the dog]*. The examples illustrate two different uses of 'be' as an auxiliary. In the first 'be' is part of the progressive aspect, used with "-ing" on the verb, and in the second it is part of the passive, as indicated by the perfect participle of a transitive verb.
    - The cat is playing
- existence, of the form "***there*** *copula noun*" *[There is a cat]*
    - *There is a mouse*
- location, of the form "*noun copula place-phrase*" *[The cat is on the mat]*;
    - *The cat is on the sofa*

If sameness of referent is the basis for truth testing and substitution, then by what grounds are all these other kinds of statements to be truth tested? Or within a system of propositions, by what rules can terms be substituted in or out of statements other than identity statements? One could postulate that assertion of identity underlies all propositions, but this would be difficult to reconcile with consensus views that explicitly treat predicates, for example as non-denoting or non-referential. And it would fly in the face of canonical distinctions between identity and predication.

A second problem is the fact that substituting referentially identical terms -even in the restricted domain of identity statements that share the same referent – can change the truth value of the statement. For example, treating as an axiom the sentence that 'Scott is the author of Waverly', and given the subsequent sentence, " George IV wanted to know whether Scott was the author of Waverly", it should be legal to substitute 'Scott' as it occurs in the first sentence for 'the author of Waverly' in the second sentence. Of course, doing so yields the statement 'George IV wanted to know whether Scott was Scott.' But George IV did not want to know whether Scott was Scott; he wanted to know whether Scott was the author of Waverly. So in a very real sense the substitution changed not only the sense or meaning of the statement, but also its truth value. For as a statement of fact it was true that George IV wanted to know whether Scott was the author of Waverly. But as a statement of fact, it was also false that he wanted to know whether Scott was Scott. How can this be? How can a valid substitution change the truth of a proposition? Sense, which mirrored the being of the terms, naturally changed with the substitution of different terms having the same referent. But identity-based truth should remain constant across all substitutions of co-referencing terms.

The third problem is that even looking at only those statements that are supposedly 'identity statements', and just focusing on the seemingly non-problematic cases there within, *identity* is more of a metaphysical than an operational concept. It may serve as a conceptual backdrop for thinking about meaning and reference. But there is no way to test, verify or otherwise know the identity of anything. Identity is a red herring.

For example, let's take up again the well trodden "Scott is the author of Waverly.". The supposed identity between the referent of the term 'Scott' and the referent of the terms 'the author of Waverly' occurs outside of any linguistic, symbolic or representational context. This may be fine for starters, but how could such a statement be evaluated or tested without

internalizing those referents? At some point 'the author of Waverly' needs to translate into 'Scott' so that an expression having roughly the following form can be tested.

Test whether the value on the left hand side of the copula is the same as the value on the right hand side

Left hand side: Scott
Right hand side: Scott
Computed comparison: Sameness

Only through the execution of an operation (e.g., by computing) can 'Scott' actually be compared with 'Scott' and their sameness be ascertained. It may not be popular to talk about computation, but *a*, if not *the* power of logic is its computability (whether in the mind or in a computer). The whole computing industry, the billions of computer chips (i.e., logic circuits with their AND and OR gates) working around the planet are a testament to the computability of propositional logic. When a molecular proposition is evaluated through a logic circuit, the 1's and 0's passing thru the registers are the sole bearers of meaning. A 1 and a 1 are XORED into a 0 not because of what the 1's refer to but because of what they are (e.g., an electric charge) at that moment. Sense and referent must become one during the moment of computation. Or sense can play no role in how information is processed, and reference, regardless, still needs to merge with the symbol itself. So if truth is defined for predicate logic in terms of sameness of referent, then it must be shown how during the normal course of evaluating or testing a proposition that the referents are substituted in for the terms that denote them so the symbolic denotations can be directly compared[33].

[33] There's yet one more problem. This is that the only way that identity can be tested is through the evaluation of predicates, attributes or concepts. How would you know if some aliens had whisked away your spouse, parent or child and replaced her/him with a replica that matched every observable characteristic of which you were aware? You really couldn't. You couldn't even know if you had been whisked away while you slept and been replaced by a replica. Your replica self would think it was the 'real' you. And those predicates, attributes or concepts can be thought of as variables with values. And at the moment expressions are processed that contain those variables whatever values they have at that time are what will be processed. Their sense and referents will be one.

2.3 Conclusion

The consensus view would appear to be riddled with holes. There are problems

- With its grounding in the world because the consensus view of Logic's ultimate source of truth is something that can never be specified and so produces a contradictory philosophical underpinning,
- With its definition of a well formed proposition because the consensus view fails to distinguish well formed from illegitimate propositions and so produces paradoxical situations
- With its notion of a logical atom because the consensus view defines logical atoms in terms of propositions between which some dependencies must exist and so produces contradiction
- With its method of distinguishing kinds of truth because it fails to recognize assertion processes as the basis for such distinctions and so is incapable of distinguishing kinds of truth, and finally
- With its method of defining equivalence and supporting substitution because the consensus view
    - only applies to identity statements which are a subset of all possible statements
    - by ignoring sense as contributing in some fashion to criteria for substitution, produces contradictory, truth value altering substitutions and because it
    - provides no method for referents to actually be computed with

These are not minor problems. The existence of any one should be enough to force an honest re-evaluation of the consensus view. Taken together, they demand a new paradigm.

