On the Nature of Logical Implication

Abstract:

This paper will attempt to show how a Locator-Content (LC) specification of the material conditional can provide a 2-valued truth-functional account of logical implication that circumvents the usual pitfalls of paradox and application

Having dealt in detail with the classical (so-called "logical" and "semantic") paradoxes in prior papers, we will focus largely on the so-called "paradoxes of material implication" here. These have purported to undermine the application of the truth-functional 'if-then' to ordinary (especially casual) usage.

The ultimate purpose is to:

1.) provide a unified and consistent generation of the LC Propositional. Predicate and Resolutional calculi from one another and

2.) demonstrate how the LC Resolutional logic evolves out of the need to express whole-part transitive and non-transitive implications in the same manner that Predicate logic evolved from the need to express generalities not explicitly captured by the Propositional calculus of truth-functions.

While the truth-functional Resolutional logic is only a subset of the multidimensional LC formal syntax, it does extend the symbolic logic of implication by distinguishing in a unified and consistent formalism whole-part (mereological) from set-member deductions.

I. Introduction to the Enigma

Logic is a bit of a puzzle. For a discipline whose truths are held to be self-evident, its disciplinarians can't seem to agree on what those really are. And for a body of truth supposed to be timeless, it has seen it share of innovations in recent times. However this paper isn't about logical absolutism or relativism, both of which rely on extra-logical presuppositions. The evidentiary for self-evidence is ultimately epistemological, just as the ground for timelessness is an ontological one.

These issues are not inconsequential in the broader contest—in fact the broadest context—bearing as they do on language's relation to mind and being—one what your definition of 'I' is, and what your determination if 'is' is, so to speak.[1] But the tack taken here is that, whatever its extralogical grounds, logic follows from a (finite) set of rules; that its formal normative truths are internal to calculi generated by those rules—unlike ordinary "normal" truths whose values are external to their assertion conditions. Point being that the two modes of signifying are different, neither conceptually prior to the other, just as neither pure math nor its applications can claim priority.

These aren't uncharted waters, but nowadays there seems to be a headwind against this approach.

The ultimate aim of this paper is to further refine the distinctions of what are variously termed function/argument, set/member, predicate/subject, concept/object,

[1] I don't mean to sound so flippant. The question of where to place the ultimate, i.e. grounding, source of valuation has continually cycled between being, mind, and language (with language being the best represented in our present paradigm.

relation/relation, etc.: in sum, what were called content/locator. Then to generate from these formal system of mereological, whole/part, or multidimensional, types.

Such a system exists already in the LC Type-theory. The purpose here is to relate that concept of implication across scales, or “resolutions,” to classical (and non-classical) calculi of formal implication. And a good part at that agenda requires a rigorous, formal distinction between, well, formal and non-formal inference.

So, two themes will thread this treatment of Propositional, Predicate and what is being termed ‘Resolutional’ logic: First, that the calculi treated here—Propositional, Predicate, Resolutional—*can* be derived operationally from one another; and in such a way as to preserve a truth-functional equivalency between them.

Second, that the distinction between logical implication and other sorts of inference *is* formally drawn. Which is saying as much, and as little, as that logical implication is just part of the language game called logic. Which is to say that the distinction must be drawn *within* logic.

But the ‘meta-text,’ if you will—what should connect this white paper with previous ones—is that those focused on the LC criterion for distinguishing *singular* true-false propositions from non-propositions (i.e. sentence s belonging to non-truth-valued language games).

But, likewise, there is an LC criterion for distinguishing valid from invalid *truth functions* of LC propositions. This distinction can and will be formally maintained from the Propositional to Predicate to Resolutional calculi. This generative evolution requires a detailed exposition of its determination in Propositional logic, where it first surfaces as the connectives linking elementary/atomic propositions in complex/molecular ones: and above all, in the truth function expressing if-then assertions.

II. Propositional Logic

Philosophical logic crosses under a flag of truce the no-man’s-land between the necessity of proof and the contingency of natural language. At its best it offers terms of conciliation between the warring factions. At its worst it demands the unconditional surrender of one side or the other. Formalism insists language capitulate to its unassailable truths, natural language insists on conformity to actual usage. Nor surprisingly both ideologies are blinded to their own aporias and errors: natural language believes its mandate comes direct and unmediated from reality itself. Formal logic confuses *internal* validity with *eternal* truths.

The focus of this section will be on the original grounds for the *causa belli* of that dispute: the relation between the material conditional—p>q—and implication.

Symbology

p, q, r, etc: variable taking singular/atomic/elementary propositions (elemprops) as values

P,Q, R, etc.: variables taking complex/compound/molecular propositions (molecprops) as values.
ξ,η: variables taking any number of propositions as values.

- : not
A: and
V: or
>: if p then q
<: if q then p
= : if p then q, and if q then p
Ω ( ): general form of an operation
N ( ): joint-negation operation
→ : p implies q

II. A. Propositional Logic: The "Ordinary Conditional"

The ordinary conditional is for all its ordinariness still controversial after millennia of study. This is probably because our conventional if…then…expression has to shoulder the burden of two extremes of knowledge, from the guaranteed certainty of logic ('if p and q, and p; then q') to the 'what if' expressions of least certainty ('If p happens, then what q?').

The deceptively simple contention here is that both extremes are covered by logical forms—rightly conceived; that the "certainty" of implication is the result of a different logical syntax from "material" conditionals.[2]

"Rightly conceived" means conceived according to the LC Type Theory. That theory will here be expressed as best it can using the notation of symbolic logic. The reason is twofold: first so that the defenses between the usages of LC and classical logic symbols will be evident to logicians and systems designers already schooled in the canononical notation. (As a corollary, it will underscore how much of the difference in *meaning* between the terms of these theories is a matter of *usage*.)

Second it will make perspicuous just how far we can go with the standard notation and where new symbols and syntax are required. The ultimate aim of the paper is to express logical implication across hierarchies, and not surprisingly it is there that a new notation and/or new combinations or uses of the old will be required.

I will contend that the difference between cross-hierarchy expressions that *can* (and *should*) be expressed by logical implication, and those that *can't* (and *shouldn't*) be, parallels and derives from the difference between the modus ponens (MP) and material conditional (MC) of Propositional logic. Therefore it is appropriate that we start there.

And a good place to begin would be with the so-called 'paradoxes of material implication.' That there should *be* a paradox of the truth-functional $p \supset q$ is in some ways the real paradox: after all '^', 'V' and '~' are not problematic in the least. Yet somehow their combination into conditionals gives rise to controversy. And p ⊃ q is truth-functionally equivalent to ~*p* V *q*, and ~(p^~q) is we could dispense altogether and

[2] This last point will be more exemplified than proven since that would require its own separate paper, or even book, and is wide enough of the mark—i.e., implication across hierarchies—that it be an unwarranted distraction. Of the various species of ordinary conditionals, the two that will be discussed are the 'indicative' (if p is true, then so is q) and the 'deontic' ('If p happens, do q'). And not much of the second either except as it falls under, or alongside, the first.

express our if-thens in terms of ‘ands’ ‘ors’ and ‘nots’ that are already understood from our early grasp of language, let alone a first course in logic.[3] So what’s the deal?

First a bit of history: I would argue that we know both the date and place when and where formal logic was born.[4] It was the Fourth Century BCE when Philo of Megara wrote that ‘if p then q’ is equivalent to ‘not p or q.’[5] This was the moment when logical thinking severed its last ties with mythical thinking, leading to the slow demise of the *mythos* in so-called Western culture (to the point where ‘mythical’ has come to mean fictional, and where the ‘internal logic’ of myth is nearly unintelligible to us). But, not surprisingly, it was a leg up for the *logos* (leading to the ‘internal logic’ of logic—which is not as redundant as it sounds, since the truth of logic is internal to its calculus). And logic needed a boost.

That’s because research in both cognitive science and cultural anthropology has revealed that the brain is not naturally given to logical thinking.[6] That left to its own devices the human mind is partial to the logical fallacy known as ‘affirming the consequent’: ‘if *p* then *q*, and *q*; then *p*.’ E.g. ‘when the village is about to burn I smell smoke, and I smell smoke; therefore, the village is about to burn.’ It ain’t logical but it saved villages.

Much recent work in cultural anthropology has found this to be the norm in mythical expression.[7] So it is not surprising that there was much resistance to Philo’s formulation in his own day. What *is* surprising is that there is still resistance to it today.

II. B. The Meaning of the Logical Connectives

The whole controversy about the “meaning” of the connectives is a bit strange to me. On the one extreme you have Platonists like Frege who tend to think of them as naming relations in the world. They’re “meaning invariant.” Even Bertrand Russell spoke of them as “logical constants,” as you might the speed of light in physics. At the other extreme you have Prior who also insists on invariance: but in his case because their

[3] The inclusive ‘or’ or ‘vel’ given here is admittedly a bit trickier than the exclusive one: ‘You can have the lollipop *or* the tootsie roll. However the V just means you’ll get the lollipop *or* the tootsie roll, *or* possibly (if your insistent enough) both.

[4] To the best of my knowledge, though, this point has never been overtly stated.

[5] It is true—and significant—that Aristotle was the first to identify the two-fold nature of a proposition in his *De Interpretatione* (what we have characterized as ‘location/content’). But a *formal* relation between these was not given.

[6] Barber and Barber, 28.

[7] This is *not* meant to imply that logical thinking is *better* than mythical thinking. Each has its uses and reasons, and modes for conveying information. Although $(p \supset q); \supset p$ is a fallacy from the point of view of logic, it may prove beneficial to the culture relying on an inductive associational praxis (and, as is argued in Barber and Barber, relying on a primarily oral tradition of conveying this information.) I think Wittgenstein would agree that each is an appropriate response to its environment. In a primarily written tradition priorities change and the information can be conveyed across generations without requiring the abbreviated and condensed significations of myth. But this argument would require far too much space, and can only be given in its abbreviated and condensed signification here. But I might go out a little further on the limb and say that once logic comes into play, information is not only a response to the environment, but the environment becomes more (and more quickly) a response to it, leading eventually, for better or worse, to a world shaped by our technologies. Computers are probably the most evident exemplification of this. In fact, it seems to be the case that here the logic of the technology *is* logic, leading to an entirely different level of representation.

meanings must be extrapolated from their ordinary language usage (Prior, 1960). In between are what Susan Haack calls Deviant Logics, which are as various as the meanings they can construct for their connectives.

There is a parallel that can be drawn between these metalogical theses and the chief foundational theses of metamathematics. In (Shavel, Thomsen 1992) we argued the Platonist, Intuitionist, and Formalist schools (like their medieval counterparts, Realism, Conceptualism, and Nominalism[8]) distinguished themselves by their stance vis-à-vis the *existence* of numbers. Platonists, like Cantor, hold that numbers are pre-existent entities; Intuitionists, like Brouwer, that they are categories of the mind; and Formalists that they are formal linguistic constructs (tokens). In that paper we argued that there is another position, on that ignores the extra-mathematical disputes about the *being* of numbers in favor of a functional investigation of their meaning. In other words, the idea that numbers are determined by their use and place in calculus, whatever stance you choose to take on their ontology.

This approach we credited to Wittgenstein, at the least in its nascencey. His *Remarks on the Foundations of Mathematics* was little understood, then or now. But this notion that meaning is determined by usage is arguably the one constant of his thinking, going all the way back to the Tractatus. It's not the place to defend that idea here, again. Because regardless whether Wittgenstein in so thought, it is the grounding insight of LC-log's functional methodology: that "logical form follows function" to (mis)appropriate Mies van der Rohe.

And so the LC approach sides squarely with the idea that the meanings of the constants are determined by the truth functional calculus. Re: Prior then, the meanings are not "generalizations" from English usage, because like a sum the matrices don't require a real-world reference for their validity. Prior mistakes a general truth for the a priori. But similarly the Platonists take the a priori as the given, not as a result of the particular rules of a calculus. They confuse what is an *internal* truth with an eternal one. In between are the logics that believe we can tweak the meanings ad hoc and at will to fit a desired situation. True, you can change the rules, *almost* arbitrarily. But, when you pull at one thread much else is unwoven. You can add a third truth value and derive a complete and consistent logic. But when you do a certain indeterminacy creeps into the connectives, all the way to the roots. In some systems I V~I B T; in some it's I. Not-true is not false, and vice-versa. The meaning of '>' hangs in the balance.

So, finally to get to the point: just as in previous papers we argued that functional criteria determined how and whether a sentence fit an (elementary) propositional variable—*p*, *q*, f(x), f(x,y), etc.—so here the emphasis will be on the criteria for how and whether an assertion fits a (molecular) truth function of propositions. Elaborating these in Proplog will be a springboard for the further elaborations in Pred-and Res-logs.

Sine much confusion on the literature is, as always, a matter of terminology, I'll try to be clear here up front:

Ordinary Conditional (OC)—all the natural language usages of if…then…

Material Conditional (MC)—the truth-functional if…then…Just as only a subset of natural lang. uses of the world 'plus' are examples of the arithmetical '+,' so only a subset of the OC are examples of the material conditional '⊃.'

[8] See Quine.

Logical Inference, *Logical Implication* (LI)—deductive, x-functional consequence/conclusion entailment of propositions from others.

Material Implication—a hybrid term that usually implies the treating of MC like LI, or at least ignoring the distinction between the two. A nonsense term.

II. C. Of Matrices and Molecprops

I'm assuming at least a passing acquaintance with truth tables and the matrix method. The left-hand columns of the matrices list the propositions that are the arguments of a truth-function/molecprop. (For simplicity's sake I'll be restricting the discussion to two arguments, the elemprops *p* and *q* (see below)). The rows list every possible permutation of their combined truth and falsity (i.e., in the first row the case where *p* is true and *q* is true; in the second where *p* is true, *q* is false, etc.).

The column to the right gives the sense of the molecprop truth-function. It asserts its agreement and disagreement with each of the truth-possibilities—which is equivalent to those states-of-affairs that make it true, and those that make it false. So, for example the molecprop *M*:

p, q	M
T T	T
T F	F
F T	T
F F	T

Agrees with every row except the second, the case where *p* is true and *q* is false. It asserts (*p* A *q*), which is of course our material conditional (*p*>*q*); say, 'If it rains then the levees will be breached.' It is a contingent assertion: in the particular case where it trains and the levees aren't breached it will be false. No matter how certain one is in making a contingent assertion there is no *logical* necessity to it. (Of course from a logical point of view it isn't even necessary that the sun will rise tomorrow).

Now given the four possible combinations for the truth and falsity of two arguments *p*, *q* there are precisely 16 truth functions that can be asserted of them. (More generally for *n* elemprops there are 2^n truth-possibilities and 2^{2n} truth-functions that can be constructed.) I say precisely, because any further combinations of them—truth-functions of these molecprops, truth-functions of those truth-functions etc.—will always yield a combination of signs that is equivalent to one of these original 16. Since this has been proven elsewhere I won't take the time here. But I will give all 16 because they will be referenced throughout:

p, q	A	B	C	D	E	G	H	I	J	K	L	M	N	O	P
T T	F	F	F	F	T	T	T	F	F	F	F	T	T	T	T
T F	F	F	F	T	F	F	T	F	T	T	T	F	T	T	T
F T	F	F	T	F	F	T	F	T	F	T	T	T	F	T	T
F F	F	T	F	F	F	F	F	T	T	F	T	T	T	F	T

A (*p*A~*p*) A (*q*A~*q*) (i.e. contradiction)
B ~*p*A~*q* (neither *p* nor *q*; N(*p*, *q*))
C ~*p*A*q* (*q* and not *p*)
D *p*A~*q* (*p* and not *q*)
E *p*A*q* (*p* and *q*)
F (*p*>*q*)A(*p*<*q*); *p*≡*q* (*p* iff *q*)
G *q*
H *p*
I ~*p*
J ~*q*
K (*p*A~*q*)V (*q*^~*p*) (*p* or *q* but not both)
L ~(*p*A*q*) (not both *p* and *q*)
M ~(*p*A~*q*); *p*>*q* (material conditional)
N ~(*p*A*q*); *p*<*q* (if *q* then *p*)
O *p*V*q* (*p* or *q*)
P (pV~p) and (qV~q) (i.e., tautology)

I've shuffled the order usually given, first to highlight certain symmetries, and also to suggest an algorithmic implementation. And obviously I've reversed the usual hierarchy of tautology first, contradiction last, for reasons that will be more apparent as we treat these extreme cases. Loosely speaking, it reflects the characteristic that a tautology (P) can be inferred from any molecprop, representing and state of affairs. "A tautology follows from all propositions: it says nothing. It says nothing" (*TLP* 5.142). And likewise everything follows from a contradiction. If a contradiction of *n* elemprops is allowed into the system—as say a given premise—all other molecprops of those arguments would follow from it.[9]

The brackets connect dual or opposite propositions. A conjunction of two duals yield a contradiction, a disjunction, a tautology. Using the letters A thru P I can give a streamlined version of Leibniz' *calculus rationcinatur*:

p>q M
<u>p</u> modus ponens becomes <u>H</u>
q G

which often I'll expand to the full inference when it is helpful:

[9] This was the insight Alan Turing claimed to have triggered his idea for a mechanically implementable logic machine, the abstract model that led to the computer. The connectivity of logic gates can be thought of as maintaining validity by preventing contradiction. (Turing, whose application of this insight led to the cracking of the German War Code—the so-called Enigma Machine—credited Wittenstein with having pointed it out to him.)

M
A H
E
> G
P

P is of course tautology, and any inference with that as result is valid. The use of this method shows how easy it should be to build an automated inference calculator for same-level (logically independent) propositions.

II. D. Paradox vs. Contradiction

Despite rhetorical similarities, a paradox is not a contradiction; it is of a difference category altogether. A contradiction may well be regarded as under anathema, but it is still within the communicants of logic: ‘FFFF…” is, face it, one of the truth functions: it is the flat-out false-hood, false under all possible states of affairs; the assertion of the impossible.

-p^~p- is simply the opposite, the negation of a logical truth—pV~p—i.e. a tautology. You could say ‘the *loyal* opposition,’ or as some the ‘complement,’ because necessary and impossible truth require one another. To put it otherwise, an equivalent expression of pV~p is ~(p^~p). The Principle of Sufficient Reason, logic’s grounding rule of non-contradiction, presupposes contradiction as the other side of the coin. The obverse to its reverse.

If you couldn’t even say it, you couldn’t negate it.

But don’t get me wrong here: I’m not soft-pedaling logical contradiction. There’s a reason it’s the negation of logical truth: throw a contradiction into a deductive system and it’ll grind it to a halt. It will cease to function logically. Why? Because as Ludwig Wittgenstein pointed out in the *Tractatus-Logico-Philosophicus*, “everything follows from a contradiction.” It was this insight that Alan Turing credited as the genesis for his conception of a logical inference machine—the first computer.

This isn’t just an analogy, as you can see from

1.) p^~p
⊃2.) p

2.) p
⊃3.) pVq

1.) pA~p
> 4.) ~p

3.) pVq
^4.) ~p
q

So from any pA~p we’ve proven any arbitrary q—and r,s,t,u,v, whatever…That’s one reason why in my tally above contradiction is given first, and tautology last.

Truth functions begin with negation.

Back to the distinction I was making: a contradiction circles on itself: but more so, it snowballs, drawing all propositions into its counter-deduction. A paradox, however, is as Bertrand Russell diagnosed it: a “vicious circle,” one that has no easy way out of its counter-reasoning. It’s more like a Mobius strip: follow its reasoning from the inside and without intending you find yourself on the outside. The circle of contradiction has a simple solution. Just negate it. The vicious circle of a paradox is different. Negate ‘This sentence is false,’ and you find you’ve proven it true; affirm it then and find it is, as it claims, false, and so, true…[10]

It is odd then that the so-called ‘Paradoxes’ of Material Implication seem to act more like your garden variety contradiction insofar as implying any and all propositions. And to that degree they further the standard confusion of the two. Here they are, succinctly put, in Susan Haack’s *Philosophy of Logics*:

‘$q \rightarrow (p \rightarrow q)$’

‘$- p \rightarrow (p \rightarrow q)$’

‘$(p \rightarrow q) \vee (q \rightarrow p)$’

The background idea that makes this seem plausible is the counter-intuitive case that $p \supset q$ is true when p is false (since $p \supset q$ is equivalent to $\sim p \vee q$), or when q is true, regardless of whether p is true or false. Counter-intuitive or not, this is simply the truth-functional definition of if-then; and by itself won’t lead to difficulties. The trouble begins when logicians treat the truth of $p \supset q$ as following simpliciter from ~p and likewise from q: in other words, $\sim p ; \supset (p \supset q)$:

~p	p>q
F	T
F	F
T	T
T	T

And q: $p \supset q$:

q	p>q
T	T
F	F
T	T
F	T

from another one without setting it up as a tautology:

~p, p>q	~ p>(p>q)
F T	T
F F	T
T T	T
T T	T

and

[10] Not to imply here that there is no resolution: LC logic has already given one—but that’s not the point here.

q, p>q	q> (p>q)
T T	T
F F	T
T T	T
F T	T

Indeed, allowing automatic inferencing from a single truth function ~p would not only imply p⊃q, but p⊃r, p⊃s…; or likewise from q to p⊃q ^ r ⊃q ^ s ⊃ q, etc.—and so the conjunction of these and any other elementary proposition. What I'm getting gat of course is the confusion of logically contingent and logically necessary props, of the material conditional and modus ponens (MP) or Modus Tollens (MT), which when conflated says simply q;⊃ (qVp) ^ (qVr) ^ (qVs)…It's true, logically valid, but the point it gets across—is different than the one presented—that states (so as to imply each other). The truth of p⊃q is contingent on the truth or falsity of its arguments p, q. The propositions of logic '(p⊃q) ^ p; ⊃ q' (MP) and '(p⊃q) ^ ~q; ⊃ ~ p' (MT) are true regardless of the truth or falsity of the arguments.

If it seems a little strange that tenured logicians would still be conflating these categories, well it should. But the sources of the trouble are manifold as we'll see later. For now, suffice it to say that the confusion goes both ways: not only in treating the MC as if it were a logical inference from *p* to *q*, but likewise in treating MP and MT as truth functions on equal par with the contingent truth functions of which they're constructed.

Jackson gives a typical example of where a hypothetical syllogism for indicative conditionals "fails to preserve assertibility in the sense that both (A→B) and (B→C) are highly assertible, while (A→C) is highly unassertible."

> 'If it rained, it did not rain heavily; if it rained heavily, it rained; therefore if it rained heavily, it did not rain heavily.' (Jackson, 83)

A seeming contradiction if there ever was one. So let's try it out, treating the inference as a logical conditional, and the constituent props as elemprops; i.e. as logically independent: *p* 'it rained,' *q* 'it rained heavily' and their negations:

p	q
T	T
T	F
F	T
F	F

1.)

p ~ q	>
T F	F
T T	T
F F	T
F T	T

2.)

q p	>
T T	T
F T	T
T F	F
F F	T

3.)

1.) 2.)	^
F T	F
T T	T
T F	F
T T	T

Notice that the conjunction of the two premises is equivalent to the t-func for ~q
Now the consequent:

4.)

q ~ q	>
T F	F
F T	T
T F	F
F T	T

(i.e., also ~q). And so the logical consequent;

5.)

3.) 4.)	>
F F	T
T T	T
F F	T
T T	T

(i.e., a valid inference (i.e. ‘It didn’t rain or it didn’t rain heavily,’ *and* ‘it didn’t rain heavily or it rained’ yields the assertion that whether it rained or didn’t, it didn’t rain heavily.).).

In other words, given the premise, which is equivalent to ~q—‘it did not rain heavily’—the conclusion, ~q, follows; Or, given the premises 3.) and q; then ~q:

4a.)

3.) q	^
F T	F
T F	F
F T	F
T F	F

5a.)

4a.) ~q	>
F F	T
F T	T
F F	T
F T	T

(Because everything follows from a contradiction).

That is, given this connection of assertions about the weather, it follows that regardless of whether or not it rained, it did not rain heavily.

But wait a second: the inference holds only because we took the consequent seriously: q⊃~q, 'if it rained heavily then it did not rain heavily,' which is of course absurd—that's the point of the contradiction. That *is* the contradiction…but is it? Well, looking at the T-tables apparently it isn't. To assert q^~q *would be*—FFFF, as is easily seen. However, unintuitive as it may seem, q⊃~q is a senseful assertion. And on closer examination that isn't as ridiculous as it seemed at first blush: q ⊃~q result in ~q. That is, if its true that 'if it rained heavily then it did not rain heavily'—well then, it obviously did not rain heavily. Look to the disjunctive form: q ⊃ ~q is ~qV~q, i.e. just ~q.

And here—with senseful assertion 4.)—*is the clearest proof* of how the material conditional *is not* the same as logical inference. Compare them side by side:
1.) q⊃~q with 2.) R⊃~R, where R is a valid inference, say the tautology q⊃q:[11]

1.)

q~q	>
T F	F
F T	T
T F	F
F T	T

2.)

q q	>
T T	T
F F	T
T T	T
F F	T

R ~ R	>
T F	F
T F	F
T F	F
T F	F

The one resulting in a senseful assertion, ~q, the other in a logical contradiction.

But just another second: *so what* if the original inference proves to be valid? We still concluded that 'If it rained heavily then it did not rain heavily,' which valid or not is highly unsatisfactory. It may survive the test of truth-functionality—'q⊃~q' is equivalent to asserting ~qV~q untangling the seeming paradox—but it does so at some expense of sense. It is if anything a Pyrrhic victory, practically a vindication of the view that the t-functional ⊃ doesn't reflect any normal sense of if…then. Even at best—accepting that q⊃~q is a senseful assertion, assertion 2.), the second premise, asserts a contingent t-func. (TTFT) and misses the fact that it's a necessary relation in what circumstance could it rain heavily without raining?

[11] How is q⊃q a tautology and q⊃~q not a contradiction, but a senseful prop? Because the negation of q⊃q is ~(q⊃q)—a different t-func: and another way of seeing how Jackson's contradiction is not a logical one.

The hitch is our supposition that q and p, ‘it rained’ and ‘it rained heavily,’ were treated as logically independent: which of course they’re not. They are not just semantically intertwined, but truth-functionally as well. Later, when we come to defining resolutional inference—inference across hierarchical types, we will see that ‘it is raining heavily’ is best treated as a subtype of ‘it is raining.’ But the logical construction of cross-type relations goes all the way back to Proplog: for example the Weather Type R[12] ‘Rain” made up of say subtypes *p*, ‘drizzling,’ *q*, ‘heavily,’ and *r* ‘cats and dogs’—or for simplicity’s sake just *p* and *q*. So ‘It is raining’ cannot relate to ‘it is raining heavily’ as logically independent: in this usage R is equivalent to ‘it is drizzling’ *or* ‘it is raining heavily.’ To be able to distinguish raining heavily from raining: they would t-functionally assert the same prop.[13]

So the weather inference should really be:

1.)

(pVq) ~q	>
T F	F
T T	T
T F	F
F T	T

2.)

q (pVq)	>
T T	T
F T	T
T T	T
F F	T

The premise 3.):

1.) 2.)	^
F T	F
T T	T
F T	F
T T	T

Conclusion 4.):

q ~q	>
T F	F
F T	T
T F	F
F T	T

[12] I’m drastically simplifying the parts of a Type, as well as the type-instance relations. These will be treated in their proper place. For now I’m just trying to show how even Proplog t-funcs exhibit the sort of interconnectedness that—already by Predlog—will begin to unfold into cross-hierarchy inference.

[13] It is also arguable that there needs to be three, else R ^~q would imply p (if its raining but not heavily then it must be drizzling). But we’ll ignore that here.

Inference 5.):

3.) 4.)	>
F F	T
T T	T
F F	T
T T	T

Again a valid inference, yet far more satisfactory. First, notice that 2.) is a tautology, as we would hope to be the case. (Unlike the former, where the relation f raining heavily to raining merely disagreed with the expressed “truth possibility” that it rained heavily but didn’t rain.)

The whole premise then conjuncts an assertion that it is not raining heavily with a tautology, which, as is to be expected, yield ‘it is not raining.’ The conclusion again, $q\supset\sim q$, yields the same, and so a valid inference. But again, what about that counter-intuitive $q\supset\sim q$? As an inference, as we’ve seen, it’s a sore thumb. As a contingent assertion, not quite: like it or not, that’s the whole point of this exercise: the assertion of $q\supset\sim q$—an MC—isn’t a logical contradiction. Look at it in the disjunctive form. It becomes obvious.

But—and this is crucial, and a defining feature of the LC-log Type Theory (LCTT)—the ‘OR’ of these subtypes and/or instances is the *excluse* XOR, not the usual *vel* (V). In other words, ‘It is raining’ cannot be analyzed to “ ‘it is drizzling’ or ‘it is raining heavily’ or *both*.” It’s one or the other.[14]

1.)

p q	XOR
T T	F
T F	T
F T	T
F F	F

to

(K) p	>
F F	T
T T	T
T F	F
F T	T

2.)

q (pXORq)	>
T F	F
F T	T
T T	T
F F	T

to

(p<q)(-pV-q)	^
T F	F
T T	T
F T	F
T T	T

II. E. Using Operations to Generate Proplog

A single feature of the LC Log approach—perhaps *the* signal feature—is the generation of the Propositional, Predicate, and Resolutional calculi from one another. It is this feature that guarantees the extension (truth-functional) equivalency of each.

But first step requires we show that the originary—proplog—calculus can be operationally derived: which will attempt here. The operation used is the joint-negation N(ξ) given in Wittgenstein’s *Tractatus*—which was itself a generalization of the Sheffer-

[14] This crucial distinction is what prevents the inference (for *two* types, as here) from ‘it is not drizzling’ to ‘it is not raining heavily.’ But it also turns the ⊃ to the ≡, which sacrifices a unidirectional conditional.

stroke ‘plq’ (‘neither p nor q’) of two elemprops extended by Wittgenstein to range over l – *n* propositions. We give first a derivation of t-funcs, then of the axioms of Whitehead and Russell’s Propositional Calculus.

II. E. 1. Derivation of Truth-Functions

Here’s an example derivation of all t-funcs of two propositions using Joint Negation:

p, q	N[p,q]	NN[p,q]	N[B,p]	NN[B,p]
T T	F	T	F	T
T F	F	T	F	T
F T	F	T	T	F
F F	T	F	F	T
	[B]	[O]	[C]	[N]

N[B,q]	NN[B,q]	N[B,C]	NN[B,C]
F	T	T	F
T	F	T	F
F	T	F	T
F	T	F	T
[D]	[M]	[H]	[I]

N[B,D]	NN[B,D]	N[C,D]	NN[C,D]
T	F	T	F
F	T	F	T
T	F	F	T
F	T	T	F
[G]	[J]	[F]	[K]

N[B,K]	NN[B,K]	N[B,O]	NN[B,O]
T	F	F	T
F	T	F	T
F	T	F	T
F	T	F	T
[E]	[L]	[A]	[P]

Or again (without the shortcut of ‘NN’):[15]

N[p,q]	N[B,p]	N[B,q]	N[B,C]
[B]	[C]	[D]	[H]

N[B,D]	N[C,D]	N[C,G]	N[G,J]
[G]	[F]	[J]	[A]

[15] NN[ξ] results in the complement, or opposite t-func of N[ξ]

N[C,J] [E]	N[D,E] [I]	N[B,E] [K]	N[A,E] [L]
N[A,D] [M]	N[A,C] [N]	N[A,B] [O]	N[A,A] [P][16]

The point of this exercise is to show how easily propositional logic could be machine-implemented (if only as a diagnostic tool, though my own preference leans toward a fully automated inferencing capability, on LCLI inferencing machine.[17]

In that light let's see if we can use the same methodology to generate the Axiomatic Propositional Calculus of Whitehead and Russell's *Principia Mathematica*.

Formation Rules [the metalogic]:

I. If ξ is the result of a well-formed LC query, then ξ is well-formed.

II. If ξ is well-formed then N[ξ] is well-formed.

Cor.1. If ξ, n are well-formed, then N[ξ,n] is well-formed, etc.

Cor.2. If ξ is well-formed, then NN[ξ] is well-formed, etc.

Definitions:

Def.1. If ξ = p, N[p] = -p

Def.2. If ξ = p, q, N [p,q] = -p^-q

Cor.3. NN[p,q] = pVqn

N[see truth tables for truth-functional negations and equivalencies]

Cor.4. pVq = -p>q

Axioms:

Derivations:

1) N[p,p]

= -p^-p

2) NN[(2),p]

I. =(cor.3) (-p^-p) v p [=tautology]

=(cor.4) (pVp)>p

3) N[q]

=-q

---

16 To follow the derivations, pair the t-funcs to be jointly negated, e.g. [D,E], [p^-q, p^q]

F T
T F
F F
F F

Set negation asserts for two arguments that neither obtains, that is, agreement (T) only with those rows of double F's; in this case F, F, T, T, or I, -p.

17 Theoretically, any of these t-funcs (except fautology or contradiction) could be used to derive the others. The original Sheffer-stroke used L, -pV-q (whose second iteration is p^q rather than pVq). An interesting experiment might be to use 'exclusive or' (K, whose second iteration is p≡q). What makes this more than intellectually interesting is how it might relate to the Predlog/Reslog LC type theory, where the relation among instances of a type is, more often than not, an XOR "either x *or* y *or*z…."

4) NN[p,q]
= pVq
5) NN [(3),(4)]
= -q V (pVq) [=tautology]
II. = q>(pVq)

6) N[p,q]
7) NN[q,p]
8) NN[(6),(7)]
= -(pVq) V (qVp) [=taut]
III. = (pVq)>(qVp)

9) N[p,q]
10) NN[p,r]
11) NN[(9),(10)]
= (-p^-q) V (pVr)
12) N [q]
13) N[(12),r]
14) NN[(13),(11)]
= (q^-r) V((-p^-q)V(pVr)) [=taut]
= (-qVr)>((-p^q)V(pVr))
IV. = (q>r) >((pVq)>(pVr))

[Tautologies I.-IV. Of the derivation are the Postulate Set of *Principia Mathematicas* Propositional Calculus, the set of axioms from which the classical theorems of Proplog were deduced. (A fifth axiom was later found to be redundant.) The set has been proven to be complete and consistent.[18] The transformation rules (slightly modified from the *Principia*) are two:]

Transformation Rules:

I. From ξ, the result of substituting n for each occurrence of the same variable in ξ may be inferred (rule of substitution)

II. From ξ>n, n may be inferred (rule of modus ponens).

Here "from ξ" is a given. ξ is known to be true because the axioms are tautologies: not because they're "assumed" or "self-evident," but only because they are true under all possible truth-conditions of their constituent elemprops. So the rule of modus ponens is just the general form of a tautology expressed in conditional form. That's why *within* the calculus we can state the modus ponens as a material conditional (that is without having to add "*from* ξ *and* ξ>n…" as we would with contingent props.)[19]

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[18] This is the first time (to my knowledge) that the axioms of a Propositional Calculus were operationally derived from their constituent propositions. Not that it was too difficult: it only took 14 steps: grant me 3) and 12) (-1 as "pre-calculus") and we can whittle it down to 12: A Twelve-Step Program for logicians addicted to the axiom that the axioms of logic are somehow "prior" to its proofs.

[19] Think about it: ξ = taut., n = q. So ξ, n > = q (TTT, TFF, TTT, TFF)

This, I suspect, is the source of Russell's (and succeeding logicians') conflation of MP with the MC.[20] (Russell's system was further burdened with the obfuscation that every *atomic* proposition is always true.[21])

Let's be clear here: q does *not* follow from the MC. As a contingent truth function *nothing follows from p>q* (except itself and tautology, which follows from every t-func). Look to the matrices. There are exactly four t-funcs which can't be divided into any propositions but themselves and tautology, L, M, N, and O. What you might call the primes of the two-place t-funcs. *If nothing else that alone shows perspicuously the span of different between the MC and MP*. It is a difference of genus rather than species.

[20] Not, as Quine thought, a confusion of use and mention, but a conflation of the axiomatic proposition (our ξ) which is a tautology and a conventional one (p, which is contingent).

[21] Primarily because, given his epistemological criterion of truth, atomic props are assertions of particular acquaintance, the evidence of their truth is immediately given and so can't be doubted (just as his "logical truths" are assertions so *general* they're self-evident. I say "self-evident," like acquaintance, is an epistemological criterion because its ultimate source of valuation is in knowledge—a subjective criterion that in the European, rationalist tradition traces back to Descartes' presuppositions that the true is what "I clearly and distinctly perceive to be such." Its grounding derives from his one undoubtable evidence of a self (*ego sum*) to a self (ego cogito). That's why Russell oughtn't be lumped in with Freye, whose logical valuation is unapologetically ontological. Every proposition has one of two referents: a Platonic 1 or 0, the True or the False.

